SDAC-TR-76-2



# COMPARISON OF REGIONAL ATTENUATION & IN EUROPE AND IN THE UNITED STATES

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21 APRIL 1976

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## COMPARISON OF REGIONAL ATTENUATION IN EUROPE AND IN THE UNITED STATES

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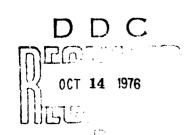
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### ABSTRACT

A maximum likelihood estimation procedure, was applied to observations of short-period P and S waves from deep earthquakes in Europe and in the United States in order to estimate regional variations in attenuation in the two regions. The separation is much less pronounced in Europe, indicating that the variability of the observations is much less than in the United States. The results indicate that attenuation effects under stations in Europe are not as important as in the United States and therefore that station magnitude biases due to attenuation are not likely to be significant.

# TABLE OF CONTENTS

	Page
ABSTRACT	2
INTRODUCTION	7
DATA	10
DATA ANALYSIS	30
CONCLUSIONS	53
ACKNOWLEDGMENTS	54
REFERENCES	55

# LIST OF FIGURES

Figure	No. Title	Page
1	Subdivision of the U.S. in high and low attenuation regions (Der, Masse and Gurski, 1975).	8
2	Geographical distribution of the stations used in Europe.	11 .
3	Geographical distribution of stations used in the	28

# LIST OF TABLES

Table	No. Title	Page
I	List of Events used for the Analysis of European Attenuation.	12
II	Amplitudes and Periods of Short-Period P and S Waves for European Stations. Exponential Distance Corrections used.	13
IIa	Reduced Variables used as Inputs in the Analysis of this report, corrected for events effects.	19
III	Maximum-Likelihood Analysis of U.S. Three-Parameter Case. [ estimated from data.	32
IV	Maximum-Likelihood Analysis of U.S. Two-Parameter Case. [ estimated from data.	34
V	Maximum-Likelihood Analysis of European Three-Parameter Case. $\sum$ estimated from data.	36
VI	Maximum-Likelihood Analysis of European Two-Parameter Case. $\sum$ estimated from data.	37
VII	Results of Maximum-Likelihood Method for the U.S. Three-Parameter Case. $\sum = I$ .	39
VIII	Results of Maximum-Likelihood Method for the U.S. Two-Parameter Case. $\sum = I$ .	41
IX	Results of Maximum-Likelihood Method for European Three-Parameter Case. $\sum = I$ .	44
x	Results of Maximum-Likelihood Method for European Two-Parameter Case. $\sum = I$ .	45
ΧI	Results of Maximum-Likelihood Method for the U.S. Three-Parameter Case. The vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).	46
XII	Results of Maximum-Likelihood Method for the U.S. Two-Parameter Case. The vector is fixed in a direction determined by the Discriminant Function in Dec. et al. (1975)	48

# LIST OF TABLES (Continued)

Table	No. Title	Page
XIII	Results of Maximum-Likelihood Method_for the European Three-Parameter Case. The $\nu$ vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).	50
XIV	Results of Maximum-Likelihood Method for the European Two-Parameter Case. The $\nu$ vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).	51
χV	Coefficients in the Discriminant Function, U.S. Case.	52

### INTRODUCTION

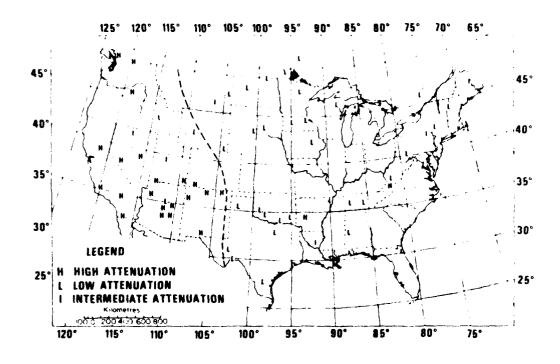
In a previous report (Der, Massé, and Gurski, 1975) we have investigated the distribution of amplitudes and periods of teleseismic short-period P and S waves over the United States using deep earthquakes. This study showed that both P and S waves have lower amplitudes in the western United States (WUS) relative to those observed in the eastern United States (EUS). The dominant periods of short-period S waves were also longer indicating that the lower amplitudes are due to the loss of high frequencies, which indicates that anelastic attenuation is the most likely explanation for the differences in amplitudes and periods. Statistical analysis of the same data (Der, Massé, and Gurski, 1975) showed that the observation points belonging to the two major regions of the USA (WUS and EUS) were well separated in both the 3 parameter space of P and S wave amplitudes and S-wave period, and the 2 parameter space of S-wave amplitude and period. A boundary between the low and high attenuation regions is defined by the data as in Figure 1.

Booth, Marshall and Young (1975) and Evernden and Clark (1970) showed that the teleseismic P magnitudes are at least .3-.4 magnitude units lower in the WUS. These determinations of magnitude differentials are more reliable than those derived from our studies quoted above since they used more events, while we were trying to establish that a qualitative difference exists also for S waves. The existence of highly attenuating regions in the mantle implies that events occurring above such portions of the mantle will have lower teleseismic magnitudes, with the consequence that the energy release of such events (or the yield of explosions) will be underestimated relative to events occurring in shields with high Q upper mantle.

Der, Z. A., Massé, R. P. and Gurski, J. P., 1975, Regional attenuation of short-perfod P and S waves in the United States, Geopys. J. R. A. S., v. 40, p. 85-106.

Booth, D. C., Marshall, P. D. and Young, J. B., 1975, Long and short period amplitudes from earthquakes in the range 0°-114°, Geophys. J. k. A. S., v. 39, p. 523-538.

Evernden, J. F. and Clark, D. M., 1970, Study of teleseismic P. II. Amplitude data, Phys. Earth. Planet. Int., v. 4, p. 24-31.



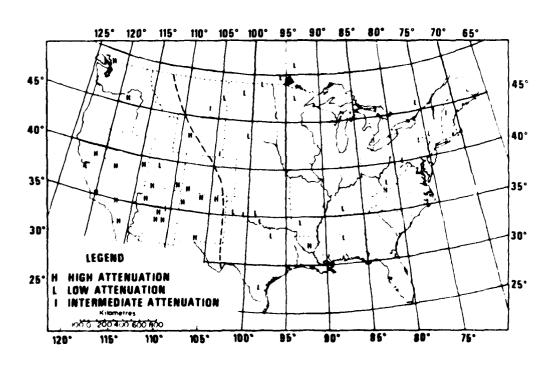


Figure 1. Subdivision of the U. S. in high and low attenuation regions (Der, Massé and Gurski, 1975).

In the case of the United States previous studies of P-wave magnitudes (Evernden and Clark, 1970) as well as studies of long-period P and S wave attenuation (Solomon and Toksoz, 1970) established an a-priori geographical subdivision for our study of short-period wave attenuation. In other regions of the world we have no prior indication of the degree of attenuation to be expected under each station. We must, therefore, devise means to order observations automatically according to attenuation. The methods used are tested in this study on the set of observations from the United States where we have two well-defined regions of low and high attenuation.

We apply the same methods to observations in Europe. Europe is an important region for observing events in the U.S.S.R., it is close to the Novaya Zemlya test site and other events in Western Russia. It is important to know how attenuation under various stations affect short-period P waves and whether or not the magnitudes determined from these waves are changed by attenuation.

Solomon, S. C. and Toksoz, M. N., 1970, Lateral variation of attenuation of P and S waves beneath the United States, Bull. Seism. Soc. Am., v. 60, p. 819-838.

Short-period P and S wave amplitudes and S-wave periods were read at the WWSSN stations AKU, ATU, AQU, COP, ESK, IST, JER, PTO, STU, TRI, UME, and VAL (Figure 2) for eleven deep focus events. Epicentral data for these events are given in Table I. The amplitudes were corrected for distances according to the procedures described in Der, Masse and Gurski (1975). It was originally intended to use the same events for an investigation of absorption in Asia, however the Asian incidence angles were shallow; and the distance corrections required were so large that if incorrect they would dominate the conclusions. We therefore confined the analysis to Europe. Unfortunately, it was not possible to find deep focus events at significantly different azimuths as in the case of the United States. All of our events are in the western circumpacific belt. The short-period system response is also slightly different from the LRSM instrumentation utilized in the previous studies. Following the procedures in Der, Massé and Gurski (1975) we defined the logarithms of P and S wave amplitudes and the S-wave period in seconds as variables and estimated the overall mean and the means for each event which were subtracted from all the observations. The reduced measurements are given in Table II, these were used for further analysis. Since no a-priori regional subdivision were available for Europe, we could not test for the significance of regional differences as in the U. S. case. We have used the United States data, which separates into two fairly well-defined populations, as a control data set and applied all the procedures used in this report to U.S. data to test their effectiveness. The two sets of data are different in the number of events and stations analyzed. The United States data includes only five events and many stations (Figure 3), while the European data has eleven events and fewer stations. Thus we have few multiple observations at a given station in the United States, while we have many in Europe.

We shall analyze two and three parameter cases as in our previous studies. The two parameter case (S-wave data only) contains more station-event pairs than the three-parameter case because P-amplitude readings were not available at several stations. We have removed readings from the U.S. data previously



Figure 2. Geographical distribution of the stations used in Europe.

TABLE I List of Events used for the Analysis of European Attenuation.

Region	Hokkaido, Japan	Sea of Ukhotsk	NW Kurile Islands	South of Honshu, Japan	Hokkaido, Japan	Sea of Japan	Sakhalin Island	Sea of Okhotsk	Sea of Okhotsk	Sea of Okhotsk	Kamchatka
Magnitude	6.2	5.6	5.5	5.8	4.0	6.8	5.9	0.0	5.7	6.1	5.7
Depth	180	453	476	349	204	417	344	645	580	544	409
Long.	165.3E	146.7E	152.5E	137.7E	143.2E	134.6E	142.5E	151.6E	151.4E	150.9E	156.3E
Lat.	44.2N	48.2N	S2.2N	32.9N	45.0N	38.3N	46.3N	52.4N	52.2N	51.7N	54.9N
Time	22:34:24.3	06:29:53.1	12:53:46.9	12:08:01.5	07:02:04.4	19:25:27.2	13:32:05.2	17:46:09.0	07:52:27.9	21:58:05.4	04:06:50.4
Date	25 Oct 65	22 Nov 66	12 Oct 67	28 Feb 68	19 Jan 69	31 Mar 69	18 Dec 69	30 Aug 70	05 Sep 70	29 Jan 71	27 May 72
Event	<b>~</b> -4	7	3	4	2	9	r~	œ	6	10	11

TABLE II

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ°
			Event 1			
AKU	343.2	1.4	19.4	1.4	2	70.42
ATU	138.2	1.0	157.3	2.3	2	91.13
COP	413.1	1.0	166.3	2.0	3	77.87
ESK	122.8	0.7	129.6	2.2	3	80.38
IST	89.2	1.1	140.5	2.4	2	86.49
JER	149.4	1.0	398.3	3.1	2	91.75
STU	121.5	0.8	210.3	3.3	3	85.07
TOL	627.2	1.5	<198.6	0.0	0	95.75
TRI	0.0	0.0	49.6	1.5	3	86.98
UME	168.5	1.0	86.6	1.6	3	68.84
VAL	250.8	1.1	82.3	1.9	3	84.16
			Event 2			
AKU	16.5	1.2	66.0	2.3	3	65.84
ATU	19.4	1.3	23.8	1.3	3	80.35
COP	12.5	1.0	32.3	1.5	2	69.63
ESK	17.6	1.3	<54.9	0.0	0	73.80
IST	9.5	1.2	39.5	1.9	3	75.47
JER	12.4	1.1	76.5	2.0	3	79.60
STU	27.2	1.2	< 5.2	0.0	0	76.64
TOL	21.9	1.4	<24.3	0.0	0	88.54
TRI	10.3	1.1	<14.4	0.0	0	77.82
UME	23.2	1.1	46.0	1.4	3	60.68
VAL	31.9	1.2	<62.0	0.0	0	78.31

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TABLE II (Continued)

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ°
			Event 3			
AKU						
ATU	100.1	1.0	8.2	1.2	1	79.73
AQU	73.1	1.0	44.7	1.8	3	79.38
COP						
ESK	57.5	1.0	0.0	0.0	0	70.97
IST						
JER	0.0	0.0	7.2	1.0	2	80.07
PTO	15.3	1.0	<109.3	0.0	0	85.60
STU	7.5	1.0	< 9.2	0.0	0	74.66
TRI	59.2	1.0	8.6	1.0	3	77.82
UME	38.7	1.0	2.9	1.0	2	60.68
VAL						
			Event 4			
AKU	24.1	1.3	152.5	2.5	3	79.96
ATU	31.6	1.3	<166.6	0.0	0	86.50
COP	15.0	0.8	131.4	2.0	3	80.19
ESK	6.8	1.0	< 13.6	0.0	0	85.97
IST						
JER	42.6	1.0	22.0	1.2	3	82.64
STU	6.9	1.0	122.4	2.4	2	86.63
TRI						
UME	16.1	0.9	57.5	1.8	3	58.49
VAL						

TABLE II (Continued)

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ°
			Event 5			
AKU	609.0	1.0	276.9	2.5	3	68.70
ATU	124.4	0.9	1000.7	2.3	3	81.03
AQU	0.0	0.0	903.4	2.3	1	82.16
COL	358.4	1.0	538.8	1.4	3	71.43
ESK	0.0	0.0	1813.1	3.5	3	76.09
IST						
JER	199.4	1.0	3927.9	2.2	3	79.48
PTO	104.9	1.0	1434.7	2.9	3	90.60
STU	367.1	1.0	890.8	2.3	3	78.32
TRI	79.6	0.9	1471.3	2.5	3	79.22
UME	122.9	1.1	789.6	2.0	3	62.59
VAL	464.2	1.0	2155.2	2.6	3	80.76
			Event 6			
AKU	45.5	1.1	153.0	2.7	3	74.20
ATU	166.5	1.2	297.8	3.0	3	81.00
COP	126.3	1.0	183.9	2.0	3	74.25
ESK	77.8	1.2	215.8	2.2	3	80.02
IST	145.9	1.2	222.9	2.5	3	75.90
JER						
STU	47.7	1.0	26.4	2.2	3	80.72
TOL	42.5	1.0	66.1	2.5	2	93.46
TRI	54.2	1.0	56.7	2.0	2	80.92
UME	68.4	0.7	83.1	1.5	3	65.85
VAL	47.6	1.0	<363.2	0.0	0	85.04

TABLE II (Continued)

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ°
			Event 7			
AKU	389.0	1.0	< 49.9	0.0	0	67.34
ATU	25.0	1.0	62.7	2.3	2	79.76
AQU	0.0	0.0	56.9	1.7	3	80.81
COP	70.3	1.0	<302.9	0.0	0	70.05
ESK						
IST	12.9	1.0	14.3	1.5	1	74.08
JER	23.1	0.8	78.4	2.0	2	78.35
PTO	34.4	0.8	< 29.6	0.0	0	89.21
STU	33.3	0.8	26.4	2.5	2	76.95
TOL	91.3	0.6	147.5	2.5	3	89.13
TRI	0.0	0.0	0.0	0.0	2	77.87
UME	59.1	1.0	67.9	1.5	3	61.21
VAL	89.2	1.2	< 59.8	0.0	0	79.37
			Event 8			
AKU						
ATY	669.7	0.7	3526.7	1.6	3	79.23
COP	0.0	0.0	1278.0	1.9	3	67.12
ESK	0.0	0.0	498.2	1.4	3	70.64
IST	201.1	0.8	1720.8	2.1	3	74.50
JER						
STU	0.0	0.0	1076.4	2.2	2	74.25
TOL	0.0	0.0	909.3	2.4	3	85.69
TRI	0.0	0.0	525.9	1.8	2	75.78
UME						
VAL	0.0	0.0	798.2	2.2	3	74.92

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TABLE II (Continued)

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ°
			Event 9			
AKU	57.6	1.0	43.3	0.0	0	62.17
ATU	341.8	1.0	161.0	1.9	2	79.31
COP	233.1	1.0	0.0	0.0	0	67.26
ESK	60.9	0.7	28.2	1.5	3	70.80
IST	54.9	0.8	43.1	1.8	3	74.57
JER	0.0	0.0	312.0	2.4	2	79.54
STU	218.5	1.0	< 33.2	0.0	0	74.38
TOL	88.9	1.1	0.0	0.0	0	85.84
TRI	0.0	0.0	29.0	1.4	2	75.90
UME	0.0	0.0	35.9	1.7	1	58.22
VAL	503.4	1.5	112.0	2.0	3	75.09
			Event 10			
AKU	115.0	0.8	184.3	3.2	3	62.64
ATU						
COP	624.9	1.0	274.5	1.7	3	67.59
ESK	92.9	0.7	132.9	1.4	3	71.20
IST	98.2	1.0	125.6	1.6	2	74.74
JER	0.0	0.0	532.7	1.8	3	79.62
PTO	94.5	1.3	74.7	2.0	2	85.83
STU	346.6	1.0	256.0	2.8	3	74.70
TRI	0.0	0.0	147.8	2.5	1	76.18
UME	0.0	0.0	174.2	1.3	3	58.56
VAL						

TABLE II (Continued)

Amplitudes and Periods of Short-Period P and S Waves
for European Stations. Exponential Distance Corrections Used.

STATION	P AMPLITUDE (mµ)	P PERIOD (sec)	S AMPLITUDE (mµ)	S PERIOD (sec)	No. of COMPONENTS	Δ,
			Event 11			
AKU	51.3	0.8	<28.3	0.0	0	59.66
ATU						
AQU	51.3	0.8	<28.3	0.0	0	78.08
COP	120.7	0.8	72.7	1.5	3	65.88
ESK	0.0	0.0	<37.2	0.0	0	68.87
IST	38.8	0.8	32,5	1.5	3	74.38
JER	105.2	1.0	20.9	1.9	3	80.00
STU	107.3	1.0	< 3.2	0.0	0	73.06
TRI	0.0	0.0	<10.5	0.0	0	74.86
UME						
VAL						

-18-

TABLE IIa

Variable 1 = log<sub>10</sub>(S amplitude) Variable 2 = S-wave period Variable 3 = log<sub>10</sub>(P amplitude)

U.S. 3 Parameter Case

STATION	1	PARAMETERS 2	3
APOK	-0.088	-0.308	0.159
AZTX	0.225	-0.808	0.049
BLWV	0.346	-0.592	0.510
BLWV	0.534	1.218	0.021
BLWV	-0.072	-0.737	0.493
BUQB	-0.211	-0.787	0.344
BXUT	-0.083	0.492	-0.060
CPCL	-0.294	0.730	-0.346
CVTN	-0.231	-0.837	-0.204
DHNY	-0.197	-0.699	0.529
DHNY	-0.033	0.074	0.268
DRCO	-0.807	-0.459	-0.219
DRCO	0.006	0.718	-0.423
DRCO	0.061	0.691	-0.297
DRCO	-0.229	0.030	-0.343
EUAL	0.325	-0.149	0.280
EUAL	0.248	-0.508	0.097
EMUT	0.249	0.197	0.105
FRMA	0.280	0.192	0.052
GEAZ	0.065	0.775	-0.614
GDVA	0.206	0.480	-0.004
GIMA	0.496	-0.449	0.125
GPMN	0.777	-0.025	0.788
GVTX	0.535	-0.259	0.375
GVTX	-0.191	0.001	0.114
GVTX	0.560	-0.670	0.425
нвок	0.327	-0.370	0.313
HHND	0.151	-1.182	0.457
HNME	-0.079	-0.149	0.409
HNME	-0.093	0.024	0.213
KMCL	-0.524	0.118	-0.332
KMCL	-0.503	0.184	-0.330
KNUT	-0.354	0.130	-0.378
LCNM	-0.254	-0.525	-0.545
LCNM	-0.340	0.384	-0.380
LCNM	-0.435	0.257	-0.462
LCNM	-0.413	-0.437	-0.188

TABLE IIa (Continued)

Variable 1 =  $log_{10}(S \text{ amplitude})$ Variable 2 = S-wave period

Variable  $3 = \log_{10}(P \text{ amplitude})$ 

# U.S. 3 Parameter Case (Continued)

	PARAMETERS		
STATION	1	2	3
LGAZ	0.081	1.241	-0.087
LGAZ	-0.131	0.124	0.216
LSNH	0.384	-0.316	0.038
LSNH	-0.380	-0.276	-0.284
MNNV	-0.930	0.108	-0.537
MNNV	-0.091	0.697	-0.513
MPAR	-0.633	-0.703	0.127
NLAZ	0.031	0.758	-0.165
PIWY	0.002	-0.116	-0.320
PMWY	-0.137	-0.470	-0.277
RYND	0.650	-0.276	0.614
RYND	-0.003	-0.608	0.088
SGAZ	-0.238	0.508	-0.340
SJTX	0.216	-0.137	0.465
SKTX	0.323	-0.058	0.236
SNAZ	0.189	0.724	-0.171
TFCL	-0.015	0.297	-0.156
TUPA	0.151	0.063	0.199
WINV	-0.115	0.992	-0.096
WOAZ	-0.083	0.124	0.051
EBMT	0.029	-0.309	0.138
JELA	0.208	0.301	-0.205
RKON	0.155	-0.759	0.316
RKON	-0.216	-0.909	0.099

TABLE IIa (Continued)

Variable 1 = log<sub>10</sub>(S amplitude) Variable 2 = S-wave period

Variable  $3 = \log_{10}(P \text{ amplitude})$ 

# Europe 3 Parameter Case

	P.	ARAMETERS	
STATION	1	2	3
AKU	-0.765	-0.736	0.154
AKU	0.167	0.528	-0.013
AKU	0.234	0.502	-0.181
AKU	-0.560	0.284	0.493
AKU	0.240	0.773	-0.225
AKU	0.010	1.169	-0.269
ATU	-0.277	-0.472	0.058
ATU	-0.140	-0.434	0.454
ATU	-0.002	0.051	-0.197
ATU	0.529	1.073	0.339
ATU	0.153	0.521	-0.096
ATU	0.256	-0.249	0.345
ATU	0.393	-0.077	0.323
AQU	0.598	0.133	0.318
COP	0.167	-0.119	0.234
COP	-0.144	-0.305	0.101
COP	0.170	0.002	-0.388
COP	-0.271	-0.783	0.262
COP	0.320	0.106	0.218
COP	0.183	-0.231	0.466
COP	0.235	-0.330	0.068
ESK	0.059	0.081	-0.293
ESK	0.389	0.339	0.008
ESK	-0.364	-0.460	-0.426
ESK	-0.133	-0.598	-0.362
IST	-0.057	0.062	-0.252
IST	0.403	0.673	0.281
IST	-0.488	-0.229	-0.384
IST	-0.055	0.185	-0.177
IST	~0.180	-0.127	-0.471
IST	-0.157	-0.398	-0.338
IST	-0.114	-0.330	-0.425
JER	0.230	0.228	-0.139
JER	-0.607	-0.765	0.065
JER	0.592	-0.016	0.008
JER	0.250	0.271	-0.130
JER	-0.248	0.103	0.008
STU	-0.052	0.117	0.273
STU	-0.384	0.273	-0.205

TABLE IIa (Continued)

Variable 1 =  $log_{10}(S \text{ amplitude})$ Variable 2 = S-wave period Variable 3 =  $log_{10}(P \text{ amplitude})$ 

Europe 3 Parameter Case (Continued)

	I	PARAMETERS	
STATION	1	2	3
STU	-0.223	0.771	0.028
STU	0.152	0.835	0.210
TRI	-0.177	-0.634	0.226
TRI	0.166	0.284	-0.391
TRI	-0.191	0.106	-0.149
UME	-0.116	-0.453	-0.155
UME	0.010	-0.405	0.135
UME	-0.583	-0.634	0.042
UME	-0.189	-0.165	-0.358
UME	-0.105	-0.216	-0.202
UME	-0.025	-0.394	-0.048
UME	0.188	-0.229	0.278
VAL	-0.138	-0.153	0.017
VAL	0.331	0.384	0.375
VAL	0.235	0.106	0.491

TABLE IIa (Continued)

> Variable 1 =  $\log_{10}$ (S amplitude) Variable 2 = S-wave period Variable 3 =  $\log_{10}$ (P amplitude)

U.S. 2 Parameter Case

	PARAMETERS		
STATION	1	22	
APOK	-0.090	-0.316	
ARWS	-0.547	-0.445	
AYSD	0.068	-1.079	
AZTX	0.223	-0.816	
BLWV	0.274	-0.546	
BLWV	0.566	1.264	
BLWV	-0.042	-0.479	
BRPA	-0.332	-0.446	
BUC	-0.181	-0.529	
BXUT	-0.113	0.530	
BXUT	-0.085	0.484	
CPCL	-0.264	0.988	
CTOK	0.446	-0.345	
CVTN	-0.201	-0.579	
DHNY	-0.165	-0.653	
DHNY	-0.006	0.122	
DRCO	-0.879	-0.413	
DRCO	0.038	0.764	
DRCO	0.088	0.739	
DRCO	-0.199	0.288	
DUOK	0.150	-0.216	
EUAL	0.357	-0.103	
EUAL	0.246	-0.516	
FOTX	0.075	-0.546	
FOTX	0.212	0.572	
FMUT	0.279	0.455	
FRMA	0.585	0.997	
FRMA	0.278	0.184	
FSAZ	0.189	-0.079	
GEAZ	-0.007	0.821	
GDVA	0.236	0.738	
GIMA	0.528	-0.403	
GPMN	0.705	0.021	
GVTX	0.463	-0.213	
GVTX	-0.159	0.047	
<b>GVTX</b>	0.590	-0.412	
нвок	0.357	-0.112	

TABLE IIa (Continued)

Variable 1 =  $log_{10}(S \text{ amplitude})$ Variable 2 = S-wave period

Variable  $3 = \log_{10}(P \text{ amplitude})$ 

# U.S. 2 Parameter Case (Continued)

	PAR	AMETERS
STATION	1	2
HHND	0.183	-1.136
HLID	-0.953	-0.778
HLID	0.363	-0.079
HNME	-0.047	-0.103
HNME	-0.066	0.072
HNME	<b>~</b> 0.055	0.138
KMCL	-0.492	0.164
KMCL	-0.471	0.230
KNUT	0.401	0.654
KNUT	-0.256	0.564
KNUT	-0.324	0.388
LCNM	-0.326	-0.479
LCNM	-0.308	0.430
LCNM	-0.408	0.305
LCNM	-0.383	-0.179
LGAZ	0.009	1.287
LGAZ	-0.104	0.172
LSNH	0.416	-0.270
LSNH	-0.353	-0.228
MNNV	-1.002	0.154
MNNV	-0.415	-0.236
MNNV	-0.061	0.955
MMTN	-0.169	-0.379
MPAR	-0.603	-0.445
NLAZ	-0.041	0.804
PIWY	0.034	~0.070
PMWY	-0.107	-0.212
RYND	0.714	-0.196
RYND	0.679	-0.270
RYND	0.677	-0.228
RYND	-0.005	-0.616
SEMN	-0.382	<del>-</del> 0.945
SGAZ	-0.310	0.554
SJTX	0.246	0.121
SKTX	0.321	-0.066
SNAZ	0.216	0.772

TABLE IIa (Continued)

Variable 1 =  $log_{10}$ (S amplitude) Variable 2 = S-wave period Variable 3 =  $log_{10}$ (P amplitude)

# U.S. 2 Parameter Case (Continued)

	PARAMETERS		
STATION	1	2	
SSTX	0.047	-0.212	
TFCL	0.015	0.555	
TKWA	-0.608	-0.203	
TUPA	0.181	0.321	
<b>VO</b> IO	0.105	-0.646	
WINV	-0.117	0.984	
WINV	-0.315	-0.279	
WOAZ	-0.056	0.172	
EBMT	0.113	0.964	
EBMT	0.056	-0.261	
EBMT	-0.029	-0.183	
EKNV	-0.624	-0.370	
HTMN	0.284	-1.012	
JELA	0.240	0.347	
RKON	0.083	-0.713	
RKON	-0.235	- 7.603	
RKON	-0.189	-0.861	
WNSD	0.120	-1.045	

TABLE IIa (Continued)

Variable 1 =  $log_{10}(S \text{ amplitude})$ Variable 2 = S-wave period Variable 3 =  $log_{10}(P \text{ amplitude})$ 

Europe 2 Parameter Case

	PARAMETERS		
STATION	1	2	
AKU	-0.868	-0.697	
AKU	0.175	0.536	
AKU	0.311	0.593	
AKU	-0.408	0.177	
AKU	0.220	0.841	
AKU	-0.114	1.147	
ATU	-0.269	-0.464	
ATU	-0.218	-0.375	
ATU	0.150	-0.056	
ATU	0.509	1.141	
ATU	0.182	0.575	
ATU	0.537	-0.254	
ATU	0.328	-0.085	
ΔQU	0.520	0.192	
AQU	0.106	-0.023	
AQU	0.140	-0.008	
AQU	0.308	-0.076	
COP	0.064	-0.080	
COP	-0.136	-0.297	
COP	0.247	0.093	
COP	-0.119	-0.890	
COP	0.300	0.174	
COP	0.097	0.013	
COP	0.059	-0.253	
COP	0.320	-0.309	
esk	-0.044	0.120	
ESK	0.408	1.177	
ESK	0.369	0.407	
ESK	-0.313	-0.487	
ESK	-0.429	-0.468	
ESK	-0.257	-0.620	
IST	-0.049	0.070	
IST	0.383	0.741	
IST	-0.459	-0.175	
IST	0.226	0.180	

TABLE IIa (Continued)

Variable 1 =  $log_{10}$ (S amplitude) Variable 2 = S-wave period Variable 3 =  $log_{10}$ (P amplitude)

Europe 2 Parameter Case (Continued)

	PARAMETERS		
STATION	1	2	
IST	-0.245	-0.135	
IST	-0.281	-0.420	
IST	-0.029	-0.309	
JER	0.238	0.236	
JER	-0.270	-0.575	
JER	-0.530	-0.674	
JER	0.744	-0.123	
JER	0.279	0.325	
JER	0.615	0.465	
JER	0.346	-0.220	
JER	-0.163	0.124	
STU	0.100	0.010	
STU	~0.404	0.341	
STU	-0.194	0.825	
STU	0.022	0.263	
STU	0.028	0.813	
TRI	-0.195	-0.575	
TRI	0.318	0.177	
TRI	-0.211	0.174	
TRI	-0.619	-0.475	
TRI	-0.289	-0.087	
TRI	-0.417	-0.585	
TRI	-0.210	0.480	
UME	~0.219	-0.414	
UME	0.018	-0.397	
UME	-0.661	-0.575	
UME	-0.112	-0.074	
UME	0.047	-0.323	
UME	-0.045	-0.326	
UME	0.217	-0.175	
UME	-0.324	-0.235	
UME	-0.139	-0.720	
VAL	-0.241	-0.114	
VAL	-0.483	0.277	
VAL	-0.108	0.346	
VAL	0.170	0.098	

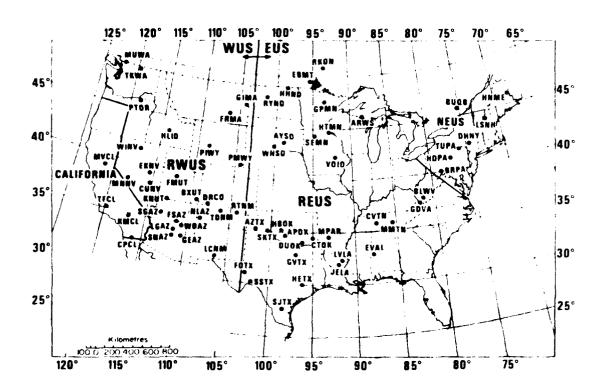


Figure 3. Geographical distribution of stations used in the U.S.

used from stations which were close to the shadow zone (PTOR and MUWA readings for the Argentine event). For the same reason all readings of European data were also eliminated from the analysis procedures which follow if the epicentral distance exceeded 85°. This restriction eliminated stations PTO and TOL where only such readings were available.

### DATA ANALYSIS

We assume that observations which are corrected for event effects at a given station i are distributed as

$$f_{1}(\vec{x}) = \frac{1}{(2\pi)^{n/2} |\vec{y}|^{1/2}} \exp \left[-\frac{1}{2} (\vec{x} - \alpha_{1} \vec{v})^{2} \sum^{-1} (\vec{x} - \alpha_{1} \vec{v})\right]$$
 (1)

where  $\vec{x}$  is the observation vector,  $\alpha_1$  is a scalar different for each station,  $\vec{v}$  is a common vector of unit length,  $\vec{v}$  is the covariance matrix of the station observations around the station mean which is assumed to be the same for all stations. The above hypothesis is based on the idea that observations at the various stations are shifted in the parameter space depending on the degree of attenuation under each station. Naturally one expects that high attenuation results in lower amplitudes and longer dominant periods yielding a shift vector corresponding to such changes. The covariance matrix  $\vec{v}$  represents scatter due to other causes.

One can search for the maximum of a likelihood function

$$L = \prod_{i,j} f_i(\vec{x}_j)$$
 (2)

where  $\mathbf{x}_{\mathbf{i},\mathbf{j}}$  is the j-th observation for station i. Substituting (1) into (2) yields a function which cannot be easily solved for the  $\alpha_{\mathbf{i}}$  and  $\nu$ . Therefore we search for the maximum by trial and error. First of all we determine  $\sum$  by summing matrices of the form

$$(\hat{x_1} - \hat{x_1}) \hat{x_1} \hat{x_2}$$

where  $\mathbf{x}_i$  is the mean of all observations at station i. Naturally we must have at least two observations from each station to estimate  $\sum$ . Subsequently we remove the mean of all observations and search with unit vectors  $\hat{\mathbf{v}}$  pointing to various directions from the origin. Since vectors in opposite directions yield identical results with the signs of  $\alpha_i$  reversed, only the upperhalf unit

sphere of the parameter space is searched. The station  $\alpha_i$  are determined subsequently by averaging the projections of  $\alpha_i$  on  $\alpha_i$  for all observations. These projections are the maximum likelihood estimates of  $\alpha_i$ . This can be shown simply by putting the derivatives of the exponent with respect to  $\alpha_i$  equal to zero (Der, Massé and Gurski, 1975). Substituting everything back into (2) through (1) yields a value of the likelihood function L.

The value of the likelihood function is mapped as a function of spherical direction angles  $\epsilon$  and  $\theta$  and a maximum is determined.

We use the U.S. data as a standard again. As in our previous study (Der, Massé and Gurski, 1975) we analyze three and two parameter cases.

Table III shows the results of the run for the U.S. three-parameter case. The vector  $\overrightarrow{v}$  at maximum likelihood does not have a component in the direction of the second parameter (S period), whereas the other two parameters (amplitudes of P and S waves) are given equal weight. This does not agree with the previous analysis of these data (Der, Massé and Gurski, 1975) where the S period was also a significant parameter, moreover it has been shown there that one can separate the populations EUS and WUS using S period alone. Nevertheless the top of the list of stations ordered on decreasing parameter  $\alpha$  contains mostly EUS stations while the bottom of list contains WUS stations. The few exceptions are identical to those found in the previous study (Der, Massé and Gurski, 1975).

The results of the run on two parameters, Table IV, also gives an almost perfect separation. The direction of the  $\vec{\nu}$  vector gives slightly more weight to the S-wave amplitudes and shows that stations with large S-wave amplitudes also have shorter S-wave periods.

Applications of the technique to the European three parameter data yields the result shown in Table V. The probe vector at maximum  $\vec{v}$  deemphasizes the third parameter (P-wave amplitude) and gives the most weight to the S-wave period. The results for the two parameter case given in Table VI (S-wave data only) give a probe vector  $\vec{v}$  at maximum L which is similar to that derived for the U.S. two parameter case. The order of stations in both cases is nearly the same, AKU and STU are at the bottom of the list. AKU is on

TABLE III  $\begin{tabular}{ll} \begin{tabular}{ll} Maximum-Likelihood Analysis of U. S. Three-Parameter \\ Case. & \sum \begin{tabular}{ll} estimated from data. \end{tabular}$ 

Station	α	Number of Events
GPMN*	1.110	1
SJTX*	.481	1
RYND*	.477	2
HECK*	.452	1
GIMA*	.439	1
BLWV*	.431	3
HHND*	.429	1
GVTX*	.428	3
SKTX*	.395	$\vec{v} = (.707, 0., .707)$
EUAL*	. 335	2
FMUT*	.250	1
TUPA*	. 247	1
FRMA*	. 234	1
DHNY*	.200	2
AZTX*	.193	1
HNME*	.159	2
GDVA*	.142	1
RKON*	.125	2
EBMT*	.118	1
BUQB*	.094	1
APOK*	.050	1
LGAZ	.027	2
SNAZ	.012	1
JELA*	.002	1
WCAZ	.023	1
ISNH*	.086	2
NLAZ	095	1
BXUT	102	1
TFCL	121	1
WINV	150	1
PIWY	225	1
PMWY	293	1

TABLE III (Continued)

Maximum-Likelihood Analysis of U. S. Three-Parameter Case. \( \sum\_{\text{estimated}} \) estimated from data.

Station	α	Number of Events
CVTN*	308	1
MPAR*	358	1
GBAZ	389	1
DRCO	398	4
SGAZ	409	1
CPCL	453	1
KNUT	518	1
ICNM	534	4
KMCL	598	2
MNNV	733	2

TABLE IV

Maximum-Likelihood Analysis of U. S. Two-Parameter

Case. Sestimated from data.

Station	ά	Number of Events
HTMN*	.752	1
HHND*	.727	1
GIMA*	.659	1
WNSD*	.627	1
RYND*	.611	4
AZTX*	.601	1
GPMN*	.600	1
AYSD*	.599	1
CTOK*	.559	1
EUAL*	.416	2
V010*	. 414	1
нвок*	.365	1
GVTX*	.355	3
SKTX*	.311	1
RKON*	. 265	3
DUOK*	.238	1 $\vec{v} = (.866,500)$
FSAZ	.203	1
BIWV*	.191	3
SJTX*	.153	1
LSNH*	.152	2
SSTX*	.147	1
SEMN*	.142	1
FOTX*	.118	2
CVTN*	.116	1
BUQB*	.108	1
APOK*	.080	1
FRMA*	.079	2
PIWY	.065	1
DHNY*	.059	2
MMTN*	.043	1
JELA*	.035	1
FMUT	.014	1

TABLE IV (Continued)

Maximum-Likelihood Analysis of U. S. Two-Parameter

Case. \( \sumeq \) estimated from data.

Station	α	Number of Events
PMWY*	.014	1
TUPA*	004	1
HLID	041	2
EBMT*	046	3
PRPA*	064	1
HNME*	066	3
WOAZ	134	1
GDVA	164	1
SNAZ	199	1
ARWS*	251	1
TFCL	264	1
MPAR*	299	1
LCNM	318	4
KNUT	319	3
BXUT	339	2
FKNV	355	1
WINV	363	2
DRCO	378	4
LGAZ	406	2
GEAZ	416	1
TKWA	425	1
NIAZ	437	1
KMCL	515	2
SGAZ	545	1
MNNV	572	3
CPCL	722	1

TABLE V  $\label{eq:maximum-Likelihood} \mbox{ Maximum-Likelihood Analysis of European Three-Parameter } \mbox{ Case. } \mbox{ } \mbox{ } \mbox{ } \mbox{ estimated from data.}$ 

Station	α	Number of Events
COP	.269	7
UME	. 247	7
AQU	.215	1
ESK	.098	4
JER	.048	5
ATU	.035	7
TRI	.035	3
VAL	.010	3
IST	057	7
AKU	418	6
STU	483	4

 $\vec{v} = (.483, -.866, .129)$ 

Station	a	Number of Events
AQU	. 208	1
COP	.196	8
JER	.161	8
ATU	.103	7
UME	.096	9
VAL	025	1
ESK	049	6
IST	049	7
TRI	117	7
STU	332	5
AKU	342	6

 $\vec{v} = (.819, -.574)$ 

iceland, a known hot spot (Morgan, 1971) with large body-wave delay times (Long and Mitchell, 1970). STU is close to the Rhinegraben, a recent rift, but it is not clear that our result is related to this fact. Stations in the Scandinavian Shield area COP and UME tend to be at the top of the list. The total variation in  $\alpha_i$  from the top to the bottom of the list is much less for two European stations than for the U.S. stations. This is especially true if AKE and STU are removed from the list. This indicates that the contrast in attenuation under the European stations investigated is much less than in the United States. Therefore, it seems that extreme attenuation effects under stations do not play the same role in Europe, not even in the Alpine tectonic bolt. It seems therefore that m<sub>b</sub>'s measured in Europe would not be biased by attenuation under the stations. This would not of course rule out attenuation effects under the sources.

The above results seem to be internally consistent but do not quite agree with some of the conclusions drawn about the relative importance of variables from the U.S. data in our previous studies (Der, Massé and Gurski). In the U.S. three-parameter case we have found that the vector  $\vec{v}$  had a zero component to the direction of the #2 (S period) variable. A likely cause of the apparent discrepancy is the scarcity of multiple observations for a given station in the U.S. data. This can cause an inaccuracy in the estimate of  $\vec{v}$ , which may be reflected in the direction of the resulting  $\vec{v}$  vector.

The assumption  $\sum$  = I where I is a unit matrix implies that the variables measured at each station are uncorrelated, a fairly reasonable assumption. Substituting I for  $\sum$  in the above analysis eliminates the effect of inaccurate f at the cost of introducing the above assumption. Results of the runs on the U.S. data are given in Table VII and VIII.

Morgan, W. J., 1971, Convection plumes in the lower mantle, Nature, v. 230, p. 42-43.

Long, R. E. and Mitchell, M. G., 1970, Teleseismic P-wave delay in Iceland, Geophys. J. R. A. S., v. 20, p. 41-48.

TABLE VII Results of Maximum-Likelihood Method for the U. S. Three-Parameter Case.  $\sum = 1$ .

Station	α	Humber of Events	
HHND*	1.260	1	
RKON*	.805	2	
BUQB*	.778	1	
AZTX*	.777	1	
RYND*	.616	2	
CVTN*	.579	1	
GIMA*	.567	1	
GPMN*	.557	1	
НВОК*	.538	1	
MPAR*	.506	1	
GVTX*	.475	3	
EUAL*	.438	$\vec{v} = (.250,866,$	
DHNY*	. 414	2 .433)	
SJTX*	.374	1	
EBMT*	.335	1	
APOK*	.314	1	
PMWY*	.253	1	
BLWV*	.247	3	
SKTX*	.233	1	
LSNH*	. 204	2	
HNME*	.167	2	
TUPA*	.069	1	
PIWY	038	1	
FMUT	063	1	
FRMA*	074	1	
WOAZ	106	1	
LCNM	191	4	
JELA*	297	1	
TFCL	328	1	
KNUT	365	1	

TABLE VII (Continued)

Results of Maximum-Likelihood Method for the U. S.

Three-Parameter Case. \[ \subseteq I. \]

Station	α	Number of Events
GDVA*	366	1
KMC L	402	2
DRCO	411	4
BXUT	473	1
L G A Z	569	2
SGAZ	647	1
SNAZ	654	1
MNNV	703	2
NLAZ	720	1
CPCL	855	1
GEAZ	921	1
WINV	929	1

TABLE VIII Results of Maximum-Likelihood Method for the U. S. Two-Parameter Case.  $\sum$  = I.

Station	α	Number of Events
HHND*	1.150	1
AYSD*	1.080	1
WNSD*	1.050	1
HTMN*	1.030	1
SEMN*	.908	1
AZTX*	.833	1
RKON*	.713	3
*010	.653	1
CVTN*	.560	1
BUQB*	.512	1
GIMA*	.448	1
BRPA*	.416	1
HLID	.402	2
ARWS*	.396	1
MPAR*	.391	$1 \qquad \vec{v} = (.087,996)$
CTOK*	.383	1
RYND*	.372	4
MMT N*	.363	1
EUAL*	.335	2
EKNV	. 315	1
APOK*	.307	1
DHNY*	.257	2
LSNH*	.251	2
DUOK*	.229	1
GVTX*	.218	3
SSTX*	.216	1
PMWY*	. 202	1
TKWA	.150	1
нвок*	.143	1
FSAZ	.096	1
SKTX*	.094	1
PIWY	.073	1

TABLE VIII (Continued)

Results of Maximum-Likelihood Method for the U. S.

Two-Parameter Case. \( \sum\_{=} = I. \)

Station	α	Number of Events
GPMN*	.041	1
FOTX*	000	2
HNME*	040	3
LCNM	-,050	4
BLWV*	056	3
SJTX*	099	1
EBMT*	168	3
WOAZ	176	1
KMCL	-,238	2
TUPA*	304	1
JELA*	324	1
MNNV	332	3
DRCO	364	4
WINV	370	2
FMUT	429	1
BXUT	513	2
KNUT	538	3
FRMA*	550	2
TFCL	551	1
SGAZ	579	1
GDVA*	714	1
LGAZ	731	2
SNAZ	750	1
NLAZ	804	1
GEAZ	818	1
CPCL	-1.010	1

A good separation is achieved in the 3-parameter case, the separation is not as good in the 2-parameter case. The direction of the vector  $\vec{v}$  for the 3-parameter case agrees much better with the findings obtained from the discriminant analysis (Der, Massé and Gurski, 1975) in emphasizing the dominance of the S-wave period and P amplitude over the S-wave amplitude as a discriminant. The two parameter case with  $\sum = 1$  completely eliminates the S-wave amplitude as a factor in separation, the same parameter was also found marginal in the discriminant analysis.

Application of the same approach to European data (Tables IX and X) again are consistent with previous results. STU and AKU again occupy the same positions while the rest of the stations change positions in the list.

The runs with  $\sum$  = I for Europe show basically the same ordering of stations but the directions of the vectors v do not agree at all with those expected on the basis of previous work on the U.S. data (Der, Massé and Gurski, 1975). On the other hand, since we already know that the European data is much less variable, one can expect that the vectors v derived from this data base are less well defined.

As a final test we fix the vectors  $\hat{\mathbf{v}}$  in the direction derived from the discriminant analysis with the direction cosines proportional to the coefficients of the individual variables in the discriminant function (Der, Massé and Gurski, 1975, Table 5); our Table XV, since these coefficients define the direction of the optimum separation in the multiparameter space.

The results of these runs are shown in Tables XI and XII for the U.S. data and Tables XIII and XIV for the European data. The order of stations is basically the same, in the European 3-parameter case IST is also at the lower end of the list with AKU and STU. The range of variation in Europe of  $\alpha$  which is the distance in the multidimensional space is about one-fourth to one-third that of the same parameter in the United States.

TABLE IX

Results of Maximum-Likelihood Method for European
Three-Parameter Case. \( \sum\_{=} = I \).

Station	α	Number of Events
UME	.345	7
COP	. 245	7
ESK	.158	4
TRI	.077	3
JER	.040	5
IST	.016	7
ATU	047	7
AQU	080	1
VAL	099	3
AKU	428	6
STU	508	4

 $\frac{1}{v}$  = (.08, .99, 0.)

TABLE X

Results of Maximum-Likelihood Method for European Two-Parameter Case. \( \sum\_{=} = I. \)

Station	α	Number of Events
UME	.360	9
COP	.193	8
TRI	.127	7
JER	.055	8
IST	.007	7
AQU	022	4
ESK	022	6
ATU	069	7
VAL	152	4
AKU	433	6
STU	451	5

 $\vec{v} = (0, 1.)$ 

TABLE XI

Results of Maximum-Likelihood Method for the U. S.

Three-Parameter Case. The vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).

Station	α	Number of Events
HHND*	.978	1
GPMN*	.906	1
RYND*	. 599	2
BUQB*	. 584	1
RKON*	.552	2
HBOK*	.535	1
SJTX*	.515	1
AZTX*	.492	1
GVTX*	.472	1
DHNY*	.437	2
EUAL*	.400	2
BLWV*	.383	3
SKTX*	.324	1
EBMT*	.267	$1 \vec{v} = (.32,47, .82)$
HNME*	.257	2
APO <b>K</b> *	.247	1
MPAR*	.232	1
TUPA*	.182	1
CVTN*	.152	1
FMUT	.073	1
FRMA*	.042	1
LSNH*	.039	2
WOAZ	043	1
PMWY	050	1
GDVA*	163	1
PIWY	207	1
JELA*	243	1
TFCL	273	1
LGAZ	276	2
BXUT	307	1

TABLE XI (Continued)

Results of Maximum-Likelihood Method for the U. S. Three-Parameter Case. The  $\overrightarrow{v}$  vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).

Station	α	Number of Events
LCNM	401	4
SNAZ	420	1
DRCO	456	4
NLAZ	482	1
KNUT	485	1
KMCL	507	2
WINV	582	1
SGAZ	594	1
CPCL	721	1
MNNV	783	2
GEAZ	847	1

TABLE XII

Results of Maximum-Likelihood Method for the U.S.

Two-Parameter Case. The  $\overrightarrow{v}$  vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).

Station	α	Number of Events
HHND*	.887	J
*////	.881	1
W2SD*	.780	1
\1\cdot\8\10*	.763	1
/T1/4*	.706	1
% i MA *	.662	1
RYND*	.604	4
€ 755 <b>K</b> *	.562	1
GPMN*	.515	1
LO10*	.505	1
EUAL*	.431	2
RR9 <b>X*</b>	. 394	3
C7.17*	.351	3
$1446  \mathrm{J}  \mathrm{K}  \star$	. 342	1
SFM <b>V*</b>	.337	1
-KTX*	. 285	$1 \rightarrow (.75,66, 0.)$
040 <b>E</b> *	.255	1
<b> </b>	. 232	1
BlagB*	. 214	1
ESAZ	.194	1
LSMH*	.188	2
< 5.7. X ★	.175	1
BLWV*	.147	3
APOK*	.141	1
MMT N*	.124	1
DHNY*	.111	2
S.71 X*	.105	1
FOTX*	.099	2
PIWY	.072	1
HI. I D	.062	2
PMWY*	.060	1
BRPA*	.046	1

TABLE XII (Continued)

Results of Maximum-Likelihood Method for the U. S. Two-Parameter Case. The  $\overrightarrow{v}$  vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).

Station	α	Number of Events
JELA*	049	1
HNME*	065	3
FRMA*	066	2
TUPA*	076	1
EBMT*	074	3
FMUT	091	1
ARWS*	116	1
WOAZ	155	1
MPAR*	158	1
EKNV	224	1
LCNM	280	4
GDVA*	310	1
TKWA	322	1
SNAZ	347	1
TFCL	355	1
WINV	394	2
KNUT	398	3
DRCO	406	4
BXUT	409	2
KMCL	491	2
LGAZ	517	2
GBAZ	547	1
NLAZ	561	1
MNNV	561	3
SGAZ	598	1
CPCL	850	1

TABLE X111 Results of Maximum-Likelihood Method for the European Three-Parameter Case. The  $\stackrel{\smile}{\nu}$  vector is fixed in a direction determined by the Discriminant Function in Der et al. (1975).

Station	.x	Number of Events
AQU	.390	1
COP	. 254	7
VAL	. 234	3
ATU	.158	7
UME	.094	7
JER	.000	5
TRI	063	3
ESK	149	4
STU	212	4
IST	225	7
AKU	239	6

 $\vec{v} = (.32, -.47, .82)$ 

TABLE XIV

Results of Maximum-Likelihood Method for the European
Two-Parameter Case. The vector is fixed in a direction
determined by the Discriminant Function in Der et al. (1975).

Station	α	Number of Events
COP	. 206	8
AQU	.187	4
JER	.154	8
UME	.130	9
ATU	.085	7
VAL	043	4
IST	044	7
ESK	048	6
TRI	090	7
STU	365	5
AKU	371	6

 $\vec{v} = (.75, -.66)$ 

3-Parameter Case	2-Parameter Case	
4.229	-	P-wave amplitude
1.644	3.439	S-wave amplitude
-2.386	-3.028	S-wave period

## CONCLUSIONS

Attempts to classify data automatically into high and low attenuation populations (or in order of increasing attenuation) were reasonably successful for the United States data which had been shown to exhibit great contrasts previously.

A maximum likelihood approach which assumed that observations at each station i are normally distributed and shifted along a common vector  $\vec{v}$  an amount  $\vec{u}_i$  proportional to the attenuation separated the U.S. data quite well. Estimates of the station data covariance matrix  $\vec{v}$  and the vector  $\vec{v}$  were somewhat unstable. The vectors  $\vec{v}$  indicated that the general direction of variation is such that while P and S wave amplitude increase the S wave period decreases (or vice versa), but the individual  $\vec{v}$  vectors vary considerably. Application of the same methods to the data from Europe indicates that the range of variation is much less in Europe, although the general direction of  $\vec{v}$  vectors is similar to the U.S. cases, the maximum likelihood approach indicated that STU and AKU are the stations under which the attenuation is the most pronounced. It seems, therefore, that attenuation under the observing stations in Europe is much less severe than in the western United States and therefore is not likely to be a significant factor in determining teleseismic magnitudes.

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## REFERENCES

- Booth, D. C., Marshall, P. D. and Young, J. B., 1975, Long and short period amplitudes from earthquakes in the range 0°-114°, Geophys. J. R. A. S., v. 39, p. 523-538.
- Der, Z. A., Massé, R. P. and Gurski, J. P., 1975, Regional attenuation of short-period P and S waves in the United States, Geophys. J. R. A. S., v. 40, p. 85-106.
- Evernden, J. F. and Clark, D. M., 1970, Study of teleseismic P. II. Amplitude data, Phys. Earth. Planet. Int., v. 4, p. 24-31.
- Long, R. E. and Mitchell, M. G., 1970, Teleseismic P-wave delay in Iceland, Geophys. J. R. A. S., v. 20, p. 41-48.
- Morgan, W. J., 1971, Convection plumes in the lower mantle, Nature, v. 230, p. 42-43.
- Solomon, S. C. and Toksoz, M. N., 1970, Lateral variation of attenuation of P and S waves beneath the United States, Bull. Seism. Soc. Am., v. 60, p. 819-838.